

# ECS 315: In-Class Exercise Solution

## Instructions

1. Separate into groups of no more than three persons.
2. The group cannot be the same as your former group.
3. Only one submission is needed for each group.
4. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
5. **Do not panic.**
6. **Only this page will be scanned and graded. Work only on this page.**

Name	ID
Prapun	555

Consider a random variable whose pmf is given by  $p_X(x) = \begin{cases} \frac{c}{x^2}, & x = -2, 1, 3, \\ 0, & \text{otherwise.} \end{cases}$

$$\frac{9+36+4}{36} = \frac{49}{36}$$

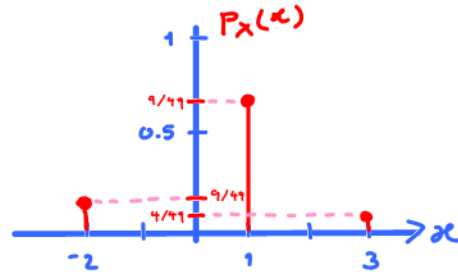
a) Find the constant  $c$ .

We know that  $\sum_x p_X(x) = 1$ . So,  $\frac{c}{(-2)^2} + \frac{c}{1^2} + \frac{c}{3^2} = 1 \Rightarrow c \left( \frac{1}{4} + 1 + \frac{1}{9} \right) = 1$

Therefore  $c = \frac{36}{49}$ .

b) Plot  $p_X(x)$ . (Recall that we use stem plot for pmf.)

$x$	$p_X(x) = \frac{c}{x^2}$
-2	$\frac{c}{4} = \frac{9}{49} \approx 0.1837$
1	$\frac{c}{1} = \frac{36}{49} \approx 0.7347$
3	$\frac{c}{9} = \frac{4}{49} \approx 0.0816$



c) Find  $P[|X^2 - 5| < 2]$ .

$x$	$ x^2 - 5 $	$ x^2 - 5  < 2$
-2	$ 4 - 5  = 1$	Yes
1	$ 1 - 5  = 4$	No
3	$ 9 - 5  = 4$	No

Therefore,  $P[|X^2 - 5| < 2] = p_X(-2) = \frac{9}{49} \approx 0.1837$

Alternatively, we can try to solve  $|x^2 - 5| < 2$  for  $x$ .

$$\begin{aligned} &\equiv -2 < x^2 - 5 < 2 \\ &\equiv 3 < x^2 < 7 \leftarrow \text{probably not too useful going beyond this} \\ &\quad \downarrow \\ &\quad \begin{cases} x^2 < 7 \equiv -\sqrt{7} < x < \sqrt{7} \\ x^2 > 3 \equiv x > \sqrt{3} \text{ or } x < -\sqrt{3} \end{cases} \\ &\quad \leftarrow \text{Only } x = -2 = -\sqrt{4} \text{ is in the above intervals.} \end{aligned}$$

d) Plot  $F_X(x)$ .

Recall that, for discrete random variable, its cdf is piecewise constant with jumps at  $x$  inside the (minimal) support. The size of the jump at  $x$  is  $p_X(x)$ .

